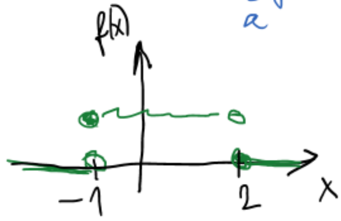
6. Nech X je spojitá náhodná premenná (NP) daná hustotou rozdelenia pravdepodobnosti
$f(x) = \begin{cases} cx + \frac{2}{9} & \text{ak } x \in \langle -1,2) \\ 0 & \text{inak} \end{cases}$
Vypočítajte: koeficient c, strednú hodnotu NP X, disperziu NP X, smerodajnú odchýlku NP X, modus NP X. Nakreslite graf funkcie hustoty rozdelenia pravdepodobnosti. Určte kvantilovú funkciu NP X, distribučnú funkciu NP X a nakreslite ich graf. Vypočítajte hodnotu dolného kvartilu. Na grafe funkcie hustoty pravdepodobnosti znázornite 9. decil, na grafe kvantilovej funkcie znázornite medián. Vypočítajte pravdepodobnosť P(X = 2), P(−2 ≤ X < 2) a P(X > 0).

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int c \cdot f(x) dx = c \cdot \int f(x) dx$

$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$

$f(x) = \begin{cases} cx + \frac{2}{9} & \text{pre } x \in \langle -1;2) \\ 0 & \text{inak} \end{cases}$



$\to c = ?$

$f(x) \geq 0$ ✓

$\boxed{\int_{-\infty}^{\infty} f(x) dx = 1}$ ✓ $\Rightarrow \underbrace{\int_{-\infty}^{-1} 0 dx}_{=0} + \int_{-1}^{2} (cx + \frac{2}{9}) dx + \underbrace{\int_{2}^{\infty} 0 dx}_{=0} = \int_{-1}^{2} (cx + \frac{2}{9}) dx = \left[c\frac{x^2}{2} + \frac{2}{9}x\right]_{-1}^{2} =$

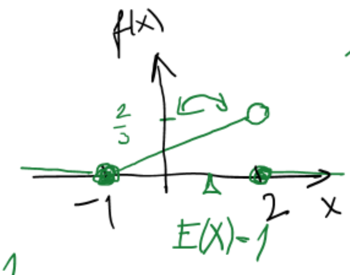
$= \left[c \cdot \frac{2^2}{2} + \frac{2}{9} \cdot 2\right] - \left[c \cdot \frac{(-1)^2}{2} + \frac{2}{9} \cdot (-1)\right] = \left[\frac{4}{2}c + \frac{4}{9}\right] - \left[\frac{1}{2}c - \frac{2}{9}\right] =$

$= \frac{3}{2}c + \frac{\cancel{6}^{2}}{\cancel{9}_{3}} = \frac{3}{2}c + \frac{2}{3} = \boxed{1} \Rightarrow \frac{3}{2}c = 1 - \frac{2}{3}$

$\frac{3}{2}c = \frac{1}{3} \Rightarrow c = \frac{2}{9}$

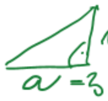
funkcia hustoty rozdel. pr.

$f(x) = \begin{cases} \frac{2}{9}x + \frac{2}{9} & \text{pre } x \in \langle -1;2) \\ 0 & \text{inak} \end{cases}$



$f(-1) = \frac{2}{9}(-1) + \frac{2}{9} = 0$

$f(2) = \frac{2}{9} \cdot 2 + \frac{2}{9} = \frac{6}{9} = \frac{2}{3}$



$v_a = \frac{2}{3}$ $S_\Delta = \frac{a \cdot v_a}{2} \Rightarrow S_\Delta = \frac{\cancel{3} \cdot \frac{2}{\cancel{3}}}{\cancel{2}} = 1$

$\to Mo(X) = ?$

není ostré l. max $\Rightarrow$ není modus $\Rightarrow$ antimodálne rozdelenie

$Mo(X) = -1$

$Mo(X) = 2$ $\Rightarrow$ bimodálne rozdelenie

$$\to E(X)=\int_{-\infty}^{\infty} x\cdot f(x)\,dx \Rightarrow \int_{-\infty}^{-1} x\cdot \underbrace{0\,dx}_{=0}+\int_{-1}^{2} x\cdot\left(\frac{2}{9}x+\frac{2}{9}\right)dx+\int_{2}^{\infty} x\cdot \underbrace{0\,dx}_{=0}=\int_{-1}^{2}\left(\frac{2}{9}x^2+\frac{2}{9}x\right)dx=$$

$$=\frac{2}{9}\int_{-1}^{2}(x^2+x)\,dx=\frac{2}{9}\left[\frac{x^3}{3}+\frac{x^2}{2}\right]_{-1}^{2}=\frac{2}{9}\left[\left(\frac{2^3}{3}+\frac{2^2}{2}\right)-\left(\frac{(-1)^3}{3}+\frac{(-1)^2}{2}\right)\right]=$$

$$=\frac{2}{9}\cdot\left[\left(\frac{8}{3}+\frac{4}{2}\right)-\left(\frac{-1}{3}+\frac{1}{2}\right)\right]=\frac{2}{9}\cdot\left(\frac{9}{3}+\frac{3}{2}\right)=\frac{2}{9}\cdot\left(3+\frac{3}{2}\right)=\frac{\cancel{2}}{\cancel{9}}\cdot\frac{\cancel{6+3}}{\cancel{2}}=1\in\langle -1;2\rangle$$

[illegible] v výpočte:

$$\to D(X)=E(X^2)-E^2(X)\Rightarrow D(X)=\frac{3}{2}-1^2=\frac{1}{2}>0\,!$$

$$E(X^2)=\int_{-\infty}^{\infty} x^2\cdot f(x)\,dx=\int_{-\infty}^{-1} x^2\cdot \underbrace{0\,dx}_{=0}+\int_{-1}^{2} x^2\cdot\left(\frac{2}{9}x+\frac{2}{9}\right)dx+\int_{2}^{\infty} x^2\cdot \underbrace{0\,dx}_{=0}=$$

$$=\frac{2}{9}\int_{-1}^{2}(x^3+x^2)\,dx=\frac{2}{9}\cdot\left[\frac{x^4}{4}+\frac{x^3}{3}\right]_{-1}^{2}=\frac{2}{9}\left[\left(\frac{2^4}{4}+\frac{2^3}{3}\right)-\left(\frac{(-1)^4}{4}+\frac{(-1)^3}{3}\right)\right]=$$

$$=\frac{2}{9}\cdot\left[\left(\frac{16}{4}+\frac{8}{3}\right)-\left(\frac{1}{4}-\frac{1}{3}\right)\right]=\frac{2}{9}\cdot\left(\frac{15}{4}+3\right)=\frac{\cancel{2}}{\cancel{9}}\cdot\frac{\overset{3}{\cancel{15+12}}}{\cancel{4}_{2}}=\frac{3}{2}$$

def:

$$(a-b)^2=a^2-2ab+b^2$$

$$D(X)=\int_{-\infty}^{\infty}(x-E(X))^2\cdot f(x)\,dx=\ldots+\int_{-1}^{2}(x-1)^2\cdot\left(\frac{2}{9}x+\frac{2}{9}\right)dx+\ldots=$$

$$=\frac{2}{9}\cdot\int_{-1}^{2}(x^2-2x+1)\cdot(x+1)\,dx=\frac{2}{9}\int_{-1}^{2}(x^3+x^2-2x^2-2x+x+1)\,dx=$$

$$=\frac{2}{9}\int_{-1}^{2}(x^3-x^2-x+1)\,dx=\frac{2}{9}\cdot\left[\frac{x^4}{4}-\frac{x^3}{3}-\frac{x^2}{2}+x\right]_{-1}^{2}=$$

$$=\frac{2}{9}\left[\left(\frac{2^4}{4}-\frac{2^3}{3}-\frac{2^2}{2}+2\right)-\left(\frac{(-1)^4}{4}-\frac{(-1)^3}{3}-\frac{(-1)^2}{2}-1\right)\right]=$$

$$=\frac{2}{9}\cdot\left[\frac{16}{4}-\frac{8}{3}-\cancel{2}+\cancel{2}-\left(\frac{1}{4}+\frac{1}{3}-\frac{1}{2}-1\right)\right]=\frac{2}{9}\left(\frac{15}{4}-3+\frac{1}{2}+1\right)=$$

$$=\frac{2}{9}\cdot\frac{15-12+2+4}{4}=\frac{\cancel{2}}{\cancel{9}}\cdot\frac{\cancel{9}}{\cancel{4}_{2}}=\frac{1}{2}$$

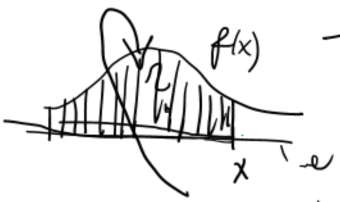
$$\to \sigma(X)=\sqrt{D(X)}\Rightarrow \sigma(X)=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$$

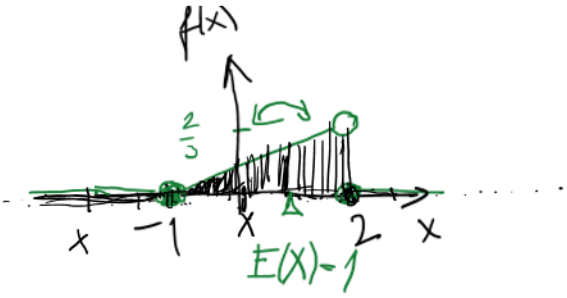
distr. f

→ $F(x) = ?$

- spoj. ✓
- neklesajúca ✓
- $H(F) = \langle 0;1 \rangle$ ✓

$f(x) = ax^2 + bx + c$ $(a>0)$ $a<0$



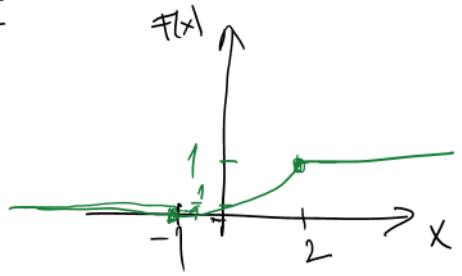


pre $\forall x \in R$: $F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$

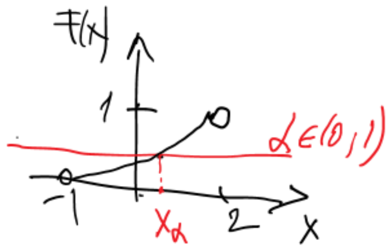
$$F(x) = \begin{cases} 0 & \text{pre } x \in (-\infty; -1\rangle \\ \int_{-1}^{x} \left(\frac{2}{9}t + \frac{2}{9}\right)dt = \frac{2}{9}\left[\frac{t^2}{2} + t\right]_{-1}^{x} = \frac{2}{9}\left[\left(\frac{x^2}{2} + x\right) - \left(\frac{(-1)^2}{2} + (-1)\right)\right] = & \\ = \frac{2}{9}\left(\frac{x^2}{2} + x - \underbrace{\left(\frac{1}{2} - 1\right)}_{= -\frac{1}{2}}\right) = \frac{1}{9}x^2 + \frac{2}{9}x + \frac{1}{9} & \text{pre } x \in \langle -1; 2) \\ 1 & \text{pre } x \in \langle 2; \infty) \end{cases}$$

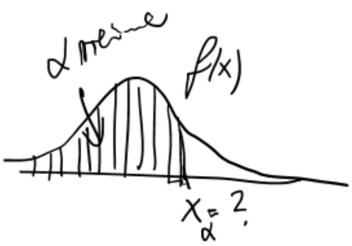
$F(-1) = \frac{1}{9}(-1)^2 + \frac{2}{9}(-1) + \frac{1}{9} = 0$

$F(2) = \frac{1}{9} \cdot 2^2 + \frac{2}{9} \cdot 2 + \frac{1}{9} = 1$



→ $F^{-1}(\alpha) = ?$ $\alpha \in (0;1)$

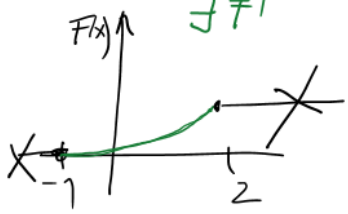




$F(x_\alpha) = \alpha \Rightarrow x_\alpha = F^{-1}(\alpha)$

α-kvantil

$F(x) = \frac{1}{9}x^2 + \frac{2}{9}x + \frac{1}{9}$ je prostá ⇒ $\exists F^{-1}$



$D(F) = (-1;2) = H(F^{-1})$

$H(F) = (0;1) = D(F^{-1})$

$\alpha = \frac{1}{9}x_\alpha^2 + \frac{2}{9}x_\alpha + \frac{1}{9}$

$\frac{1}{9}x_\alpha^2 + \frac{2}{9}x_\alpha + \frac{1}{9} - \alpha = 0 \quad / \cdot 9$

$\boxed{1} \cdot x_\alpha^2 + \boxed{2} x_\alpha + \boxed{1 - 9\alpha} = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

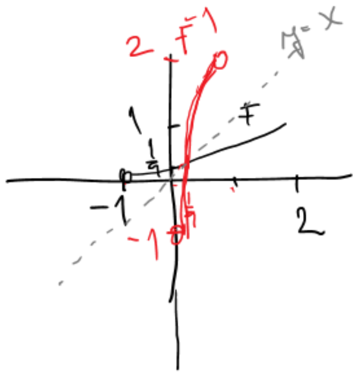
$x_{\alpha_{1,2}} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (1 - 9\alpha)}}{2 \cdot 1}$

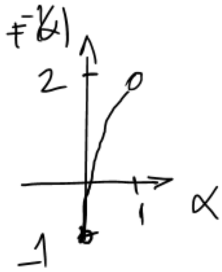
$x_{\alpha_{1,2}} = \frac{-2 \pm \sqrt{4 \cancel{-4} \cancel{+4} + 36\alpha}}{2} = -1 \pm 3\sqrt{\alpha}$

$\alpha = 0: \ -1 \pm 3\sqrt{0} = -1$

$\alpha = 1: \ -1 \pm 3 \cdot \sqrt{1} = -1 \pm 3$

$F^{-1}(\alpha) = -1 + 3\sqrt{\alpha}$; $\alpha \in (0;1)$



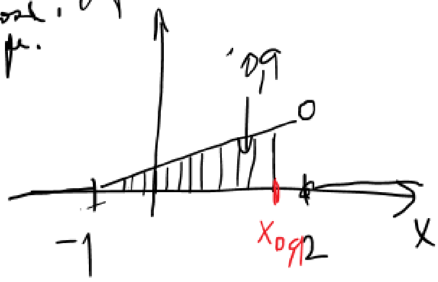


$\to X_{0,25} = F^{-1}(0{,}25) = \left|-1+3\sqrt{\alpha}\right| = -1+3\sqrt{0{,}25} = 0{,}5$

$X_{0,9} = F^{-1}(0{,}9) = -1+3\sqrt{0{,}9}$

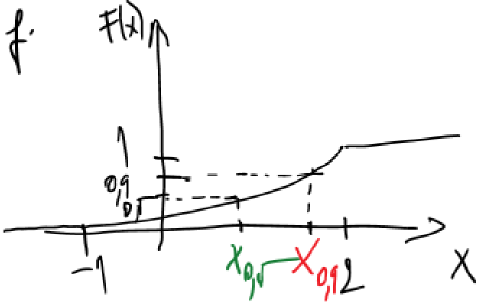
$X_{0,7} = F^{-1}(0{,}7) = -1+3\sqrt{0{,}7}$

f. hustoty rozd. pr.



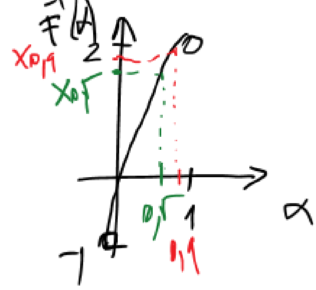
9. decil $\Rightarrow$ $X_{0,9}$

distr. f.



medián $\Rightarrow$ $X_{0,5}$

kvantil. f.

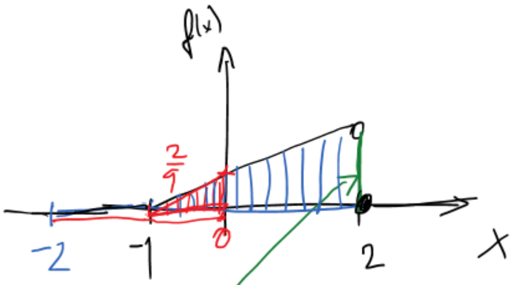


$X_{0,5} = F^{-1}(0{,}5)$

$X_{0,9} = F^{-1}(0{,}9)$

$$f(x)=\begin{cases}\frac{2}{9}x+\frac{2}{9} & \text{pre } x\in\langle -1;2)\\ 0 & \text{inak}\end{cases}$$

$$P\in\langle 0;1\rangle$$



$P(X=2)=$! [illegible]

$$\int_2^2 0\,dx=0$$

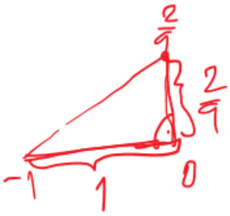
$$P(X=0)=\int_0^0\left(\frac{2}{9}x+\frac{2}{9}\right)dx=\left[F(x)\right]_0^0=F(0)-F(0)=0$$

$$P(-2\leq X<2)=\int_{-1}^{2}\left(\frac{2}{9}x+\frac{2}{9}\right)dx=1$$

$$P(-2\leq X\leq 0)=\int_{-1}^{0}\left(\frac{2}{9}x+\frac{2}{9}\right)dx=\left[\frac{2}{9}\frac{x^2}{2}+\frac{2}{9}x\right]_{-1}^{0}=0-\left(\frac{(-1)^2}{9}+\frac{2}{9}(-1)\right)=-\left(\frac{1}{9}-\frac{2}{9}\right)=\frac{1}{9}$$

$$P(-2\leq X\leq 0)=\underbrace{\int_{-2}^{-1}0\,dx}_{=0}+\int_{-1}^{0}\left(\frac{2}{9}x+\frac{2}{9}\right)dx$$

$$F(x)=\begin{cases}0 & x\in(-\infty;-1)\\ \frac{1}{9}x^2+\frac{1}{9}x+\frac{1}{9} & x\in\langle -1;2)\\ 1 & x\in\langle 2;\infty)\end{cases}$$



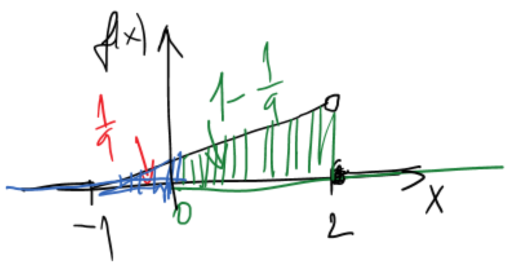
$$S_\Delta=\frac{a\cdot v_a}{2}\Rightarrow S_\Delta=\frac{1\cdot\frac{2}{9}}{2}=\frac{1}{9}$$

$$F(0)=\frac{1}{9}0^2+\frac{1}{9}\cdot 0+\frac{1}{9}$$

$$F(0)=\frac{1}{9}$$

$$P(-2\leq X\leq 0)=F(0)-F(-2)=\frac{1}{9}-0=\frac{1}{9}$$

$$P(X\geq 0)=\ ?\ =\int_0^2\left(\frac{2}{9}x+\frac{2}{9}\right)dx=\left[\frac{2}{9}\frac{x^2}{2}+\frac{2}{9}x\right]_0^2=\frac{4}{9}+\frac{4}{9}-0=\frac{8}{9}$$



$$P(X<0)=\frac{1}{9}$$